On the Smoothness and the Singular Support of the Minimum Time Function under Bracket-Generating Conditions

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Let $\Omega \subset \mathbb{R}^n$ be an open bounded set and let $X_1, \ldots, X_N$ be smooth real vector fields on an open set Ω. We assume that they satisfy the Hörmander bracket generating condition, i.e., $Lie\{X_1, \ldots, X_N\}(x) = \mathbb{R}^n$, $\forall x \in \Omega$. Here, $Lie\{X_1, \ldots, X_N\}(x)$ denotes the space of all values at x of the vector fields of the Lie algebra generated by $\{X_1, \ldots, X_N\}$. In this context we consider the minimum-time problem of minimizing the time to reach a given target set Γ following the trajectories of the Cauchy problem below

$$y'(t) = \sum_{j=1}^{N} u_j(t) X_j(y(t)), \quad t \geq 0, \quad y(0) = x. \tag{1}$$

The controls $u = (u_1, \ldots, u_N)$ take values in the n-dimensional closed ball of unit radius centered at the origin. For this problems, abnormal minimizers and singular trajectories may occur, and this destroys in general the smoothness of T.

In this talk, we investigate the (lack of) regularity of T, the properties of its singular support, and the role played by the singular trajectories. We will focus our attention on the case where the target Γ is a smooth hypersurface of $\mathbb{R}^n$.

If time permits, we will also sketch some results and open questions for the affine-control problem with drift:

$$y'(t) = X_0(y(t)) + \sum_{j=1}^{N} u_j(t) X_j(y(t)), \quad t \geq 0, \quad y(0) = x. \tag{2}$$